



Oxford Cambridge and RSA

Monday 05 October 2020 – Afternoon

A Level Further Mathematics A

Y540/01 Pure Core 1

Time allowed: 1 hour 30 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

Answer **all** the questions.

1 Find the mean value of $f(x) = x^2 + 6x$ over the interval $[0, 3]$. [2]

2 Find an expression for $1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + \dots + n(n+1)^2$ in terms of n . Give your answer in fully factorised form. [3]

3 You are given the matrix $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$.

(a) Find $\mathbf{A}^4$. [1]

(b) Describe the transformation that $\mathbf{A}$ represents. [2]

The matrix $\mathbf{B}$ represents a reflection in the plane $x = 0$.

(c) Write down the matrix $\mathbf{B}$. [1]

The point P has coordinates $(2, 3, 4)$. The point P' is the image of P under the transformation represented by $\mathbf{B}$.

(d) Find the coordinates of P' . [1]

4 In this question you must show detailed reasoning.

(a) Determine the square roots of $25i$ in the form $re^{i\theta}$, where $0 \leq \theta < 2\pi$. [3]

(b) Illustrate the number $25i$ and its square roots on an Argand diagram. [1]

5 By expanding $\left(z^2 + \frac{1}{z^2}\right)^3$, where $z = e^{i\theta}$, show that $4\cos^3 2\theta = \cos 6\theta + 3\cos 2\theta$. [5]

- 6 The equations of two non-intersecting lines, l_1 and l_2 , are

$$l_1: \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}.$$

Find the shortest distance between lines l_1 and l_2 . [5]

- 7 Prove by induction that the sum of the cubes of three consecutive positive integers is divisible by 9. [5]

- 8 (a) Using exponentials, show that $\cosh 2u \equiv 2 \sinh^2 u + 1$. [2]

(b) By differentiating both sides of the identity in part (a) with respect to u , show that

$$\sinh 2u \equiv 2 \sinh u \cosh u. \quad [1]$$

- (c) Use the substitution $x = \sinh^2 u$ to find $\int \sqrt{\frac{x}{x+1}} dx$. Give your answer in the form $a \sinh^{-1} b \sqrt{x} + f(x)$ where a and b are integers and $f(x)$ is a function to be determined. [5]

- (d) Hence determine the exact area of the region between the curve $y = \sqrt{\frac{x}{x+1}}$, the x -axis, the line $x = 1$ and the line $x = 2$. Give your answer in the form $p + q \ln r$ where p , q and r are numbers to be determined. [2]

- 9 You are given that the cubic equation $2x^3 + px^2 + qx - 3 = 0$, where p and q are real numbers, has a complex root $\alpha = 1 + i\sqrt{2}$.

- (a) Write down a second complex root, β . [1]

- (b) Determine the third root, γ . [2]

- (c) Find the value of p and the value of q . [2]

- (d) Show that if n is an integer then $\alpha^n + \beta^n + \gamma^n = 2 \times 3^{\frac{1}{2}n} \times \cos n\theta + \frac{1}{2^n}$ where $\tan \theta = \sqrt{2}$. [4]

- 10** A particle of mass 0.5 kg is initially at point O . It moves from rest along the x -axis under the influence of two forces $F_1\text{ N}$ and $F_2\text{ N}$ which act parallel to the x -axis. At time t seconds the velocity of the particle is $v\text{ ms}^{-1}$.
 F_1 is acting in the direction of motion of the particle and F_2 is resisting motion.

In an initial model

- F_1 is proportional to t with constant of proportionality $\lambda > 0$,
- F_2 is proportional to v with constant of proportionality $\mu > 0$.

- (a)** Show that the motion of the particle can be modelled by the following differential equation.

$$\frac{1}{2} \frac{dv}{dt} = \lambda t - \mu v \quad [2]$$

- (b)** Solve the differential equation in part **(a)**, giving the particular solution for v in terms of t , λ and μ . [7]

You are now given that $\lambda = 2$ and $\mu = 1$.

- (c)** Find a formula for an approximation for v in terms of t when t is large. [2]

In a refined model

- F_1 is constant, acting in the direction of motion with magnitude 2 N ,
- F_2 is as before with $\mu = 1$.

- (d)** Write down a differential equation for the refined model. [1]

- (e)** Without solving the differential equation in part **(d)**, write down what will happen to the velocity in the long term according to this refined model. [1]

- 11** A curve has cartesian equation $x^3 + y^3 = 2xy$.

C is the portion of the curve for which $x \geq 0$ and $y \geq 0$. The equation of C in polar form is given by $r = f(\theta)$ for $0 \leq \theta \leq \frac{1}{2}\pi$.

- (a) Find $f(\theta)$. [2]
- (b) Find an expression for $f(\frac{1}{2}\pi - \theta)$, giving your answer in terms of $\sin \theta$ and $\cos \theta$. [2]
- (c) Hence find the line of symmetry of C . [1]
- (d) Find the value of r when $\theta = \frac{1}{4}\pi$. [1]
- (e) By finding values of θ when $r = 0$, show that C has a loop. [2]

- 12** Show that $\int_0^{\frac{1}{\sqrt{3}}} \frac{4}{1-x^4} dx = \ln(a + \sqrt{b}) + \frac{\pi}{c}$ where a , b and c are integers to be determined. [6]

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